



QCD PHASE STRUCTURE UNDER ROTATION AND ACCELERATION

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Based on: HLC, Xu-Guang Huang, Phys.Rev.D 114 (2026) 1, 014005

HLC, Kenji Fukushima, Xu-Guang Huang, Yusuke Shimada, Yu-Han Gao, Zhi-Bin Zhu, Phys.Rev.D 114 (2026) 5, 054012

第二届旋转与磁场下QCD物质研讨会@淮南

OUTLINE

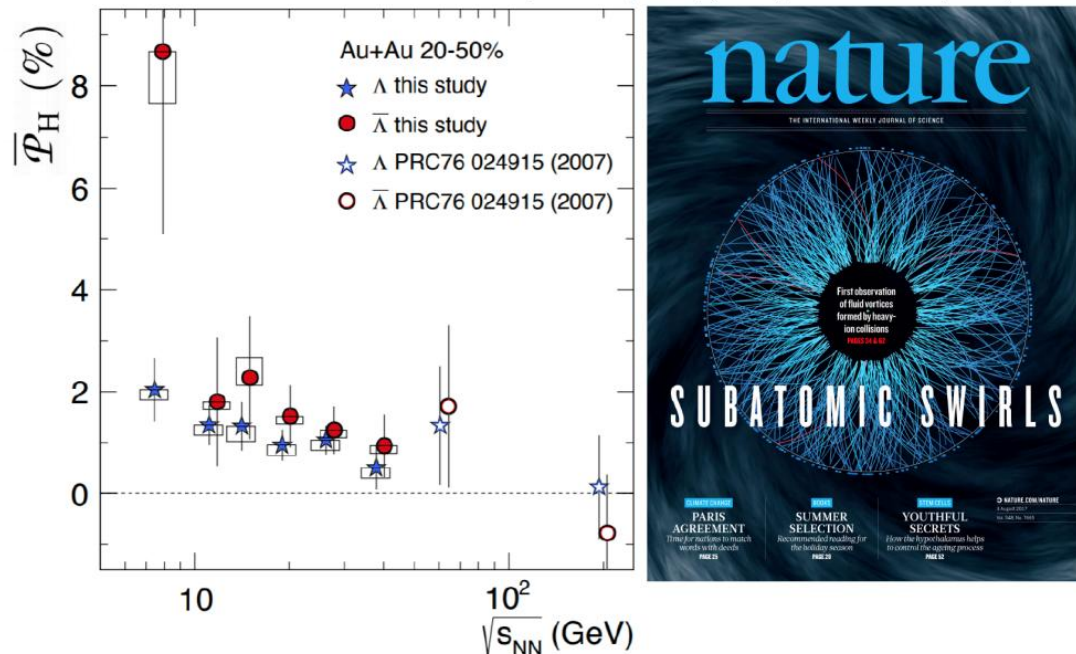
- Introduction: rotation & acceleration
- Rotation: chromomagnetic condensate Phys.Rev.D 114 (2026) 1, 014005
- Acceleration: thermodynamic subtleties
- Acceleration: Polyakov-loop potential Phys.Rev.D 114 (2026) 5, 054012
- Discussion and outlook

ROTATION AND ACCELERATION IN HIC

■ Rotation

$$\Omega = \frac{1}{2} \nabla \times v$$

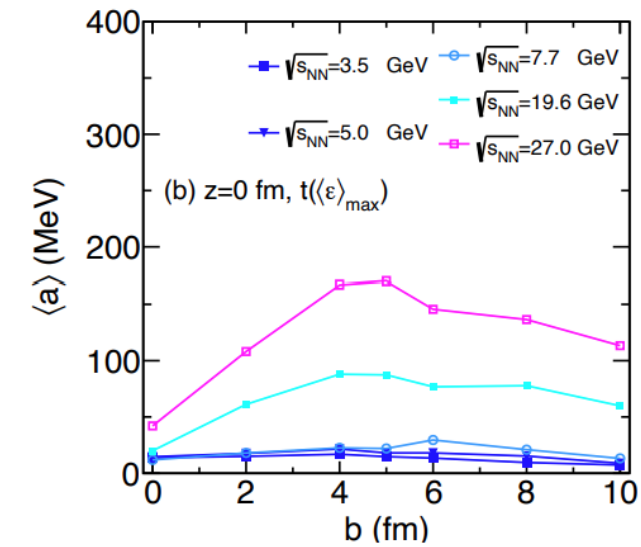
L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



most vortical fluid produced in the laboratory .

$$\omega = (P_{\Lambda} + P_{\bar{\Lambda}})k_B T / \hbar \sim 0.6 - 2.7 \times 10^{22} \text{ s}^{-1}$$

■ Acceleration $a = \frac{dv}{dt} = -\frac{1}{\epsilon + P} \nabla P$

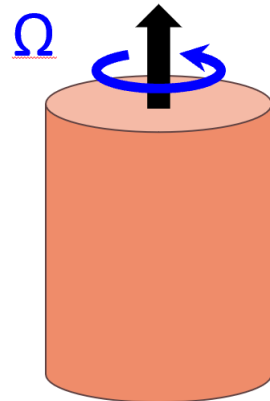


NUCL SCI TECH 37, 201 (2026)

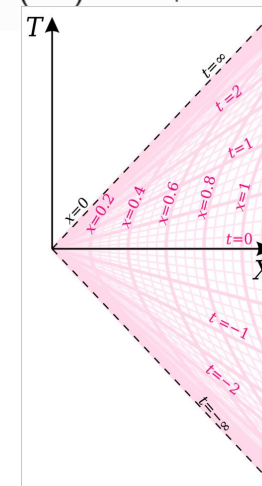
ROTATION AND ACCELERATION

	Rotation	Acceleration
Thermal vorticity	magnetic part	electric part
Symmetry	spatial rotation J_z	Lorentz boost K_z
Group structure	compact rotation $SO(2)$	noncompact boost $SO(1,1)$
Geometry	stationary, generally non-static	static Rindler geometry
Boundary	light cylinder $r = 1/\Omega$	Rindler horizon $z = 0$

$$ds^2 = -(1 - \Omega^2 r^2)dt^2 + 2\Omega r^2 dt d\phi + dr^2 + r^2 d\phi^2 + dz^2.$$



$$ds^2 = -(az)^2 dt^2 + dz^2 + dx_{\perp}^2,$$



ROTATION AND ACCELERATION

- Relativistic systems can remain in global equilibrium with acceleration and rotation
- Killing equation $\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$
- Local equilibrium density operator

$$\rho_{\text{LE}} = \frac{1}{Z_{\text{LE}}} \exp \left[- \int d\Xi_\mu \left(T^{\mu\nu} \beta_\nu - \frac{1}{2} J^{\mu\rho\sigma} \varpi_{\rho\sigma} \right) \right]$$

- Global equilibrium

$$\left\{ \begin{array}{l} \beta^\mu = (\beta, \mathbf{0}) \\ \varpi_{0i} = \beta a^i \\ \varpi_{ij} = \beta \epsilon^{ijk} \Omega^k \end{array} \right. \quad \rightarrow \quad \rho_{\text{eq}} = \frac{1}{Z_{\text{eq}}} \exp \left[-\beta (H - \mathbf{a} \cdot \mathbf{K} - \mathbf{\Omega} \cdot \mathbf{J}) \right]$$

Boost
operator

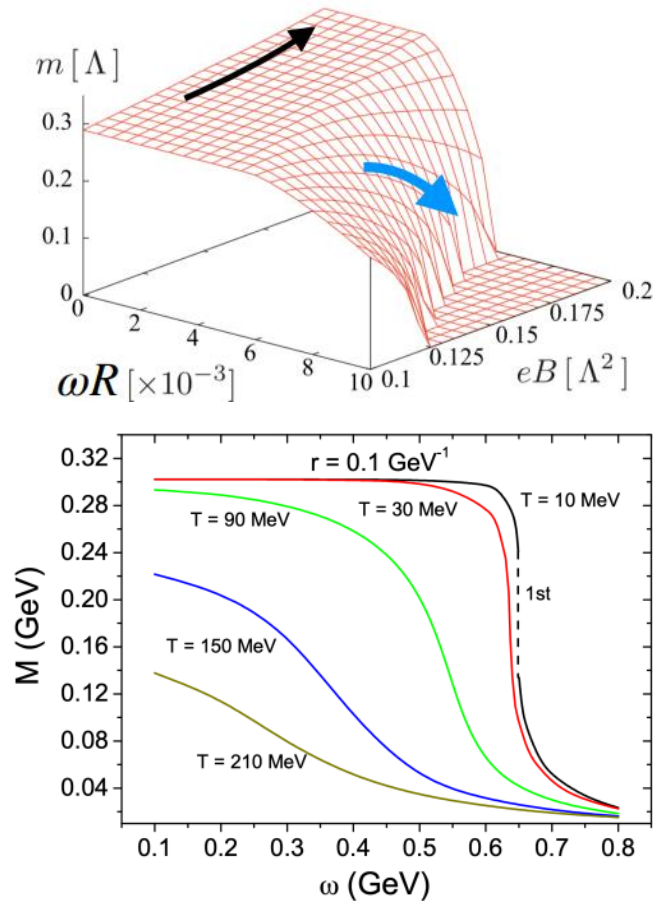
Angular
momentum

ROTATION

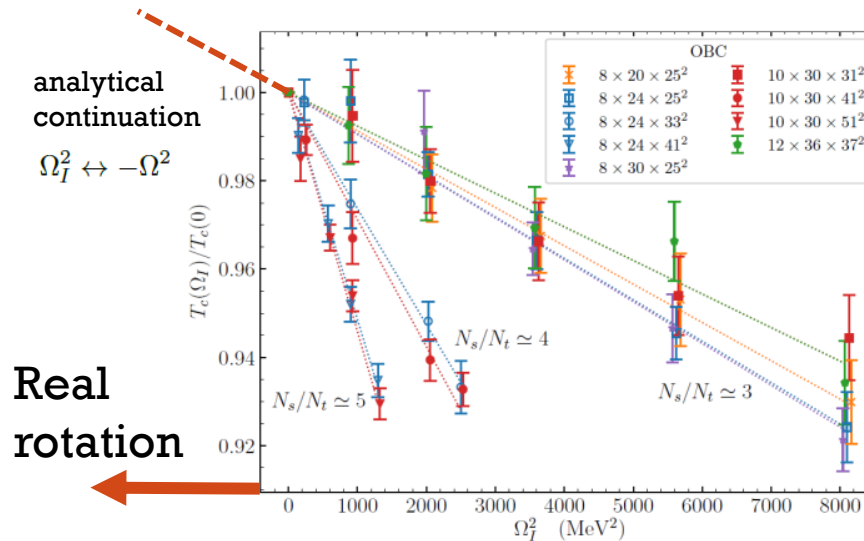
QCD PHASE DIAGRAM UNDER ROTATION

HLC, K. Fukushima, X-G. Huang, K. Mameda,
Phys. Rev. D 93, 104052 (2016)

- First studied by NJL model
- Rotation restores chiral symmetry
- However, lattice studies give opposite results!



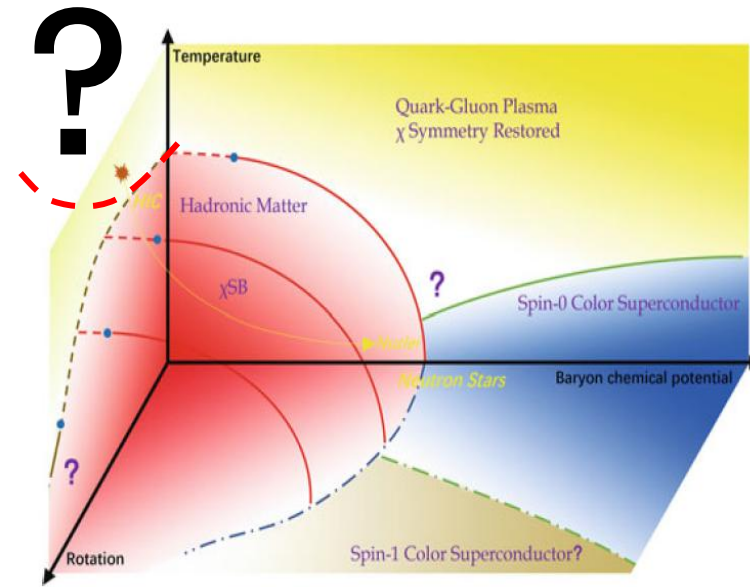
Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016)



Real
rotation

Braguta V V, Kotov A Y, Kuznedelev D D, et al. arXiv:2110.12302, 2021.

Unsolved discrepancy



GLUON

- So far, we only focused on the quark sector
- Gluon has spin one: more sensitive to rotation in principle
- It is interesting to see how rotation affects deconfinement and gluon spin

Shi Chen, Kenji Fukushima, Yusuke Shimada, Phys.Rev.Lett. 129 (2022) 24, 242002

Victor V. Braguta, Maxim N. Chernodub, Ilya E. Kudrov, Artem A. Roenko, Dmitrii A. Sychev, Phys.Rev.D 110 (2024) 1, 014511

Yin Jiang, Phys.Lett.B 853 (2024) 138655

Guojun Huang, Shile Chen, Yin Jiang, Jiaying Zhao, Pengfei Zhuang, Phys.Lett.B 862 (2025) 139274

Kenji Fukushima, Yusuke Shimada, Phys.Lett.B 868 (2025) 139716

Sheng Wang, Jun-Xia Chen, Defu Hou, Hai-Cang Ren, arXiv: 2505.15487

Takada Rin, arXiv: 2608.25849

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POLYAKOV LOOP CONDENSATE

S. Chen, K. Fukushima, and Y. Shimada
Phys.Rev.Lett. 129 (2022) 24, 242002

$$V_{eff} = \text{Polyakov Loop Diagram} - \text{Gauge Loop Diagram}$$

- Pure Yang-Mills theory under imaginary rotation
- High T: Weiss potential (one loop potential for Polyakov loop)

$$V(\phi; \tilde{\Omega}_I)|_{\tilde{r}=0} = \frac{\pi^2 T^4}{3} \sum_{\alpha} \sum_{s=\pm 1} B_4 \left(\left(\frac{\phi \cdot \alpha + s \tilde{\Omega}_I}{2\pi} \right)_{\text{mod } 1} \right).$$

- Simple replacement

$$\phi \cdot \alpha \rightarrow \phi \cdot \alpha \pm \beta \Omega_I$$

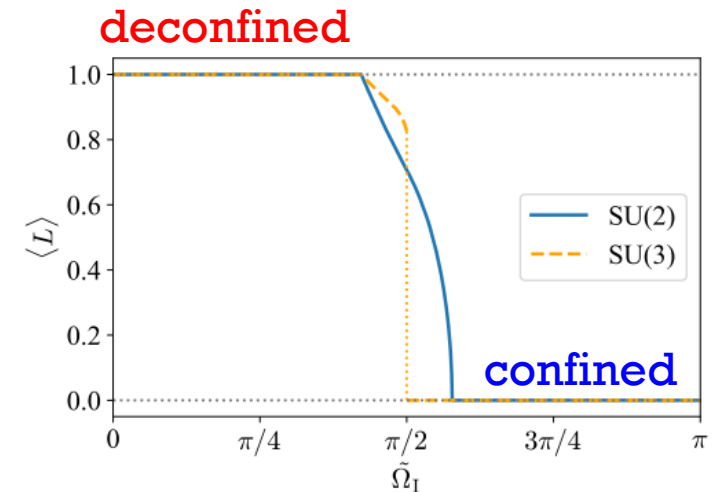
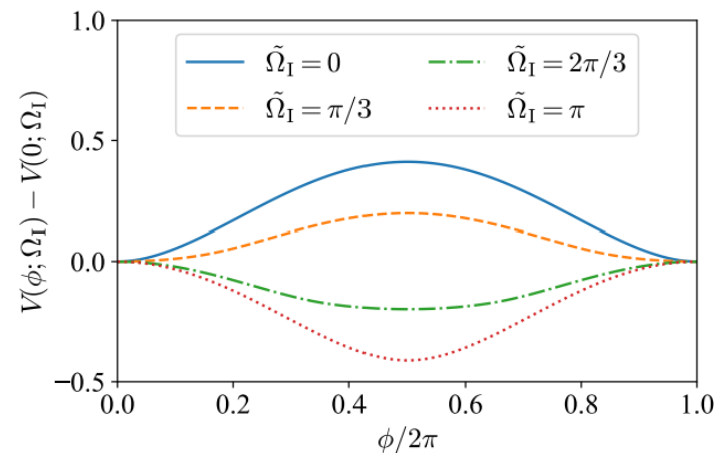
- Imaginary rotation prefer confinement

opposite
to lattice



analytical
continuation

Real rotation prefer
deconfinement



INCLUDING QUARKS

- Identity

$$e^{2\pi i \hat{J}_z} = (-1)^{\hat{F}} = e^{i\pi \hat{Q}}.$$

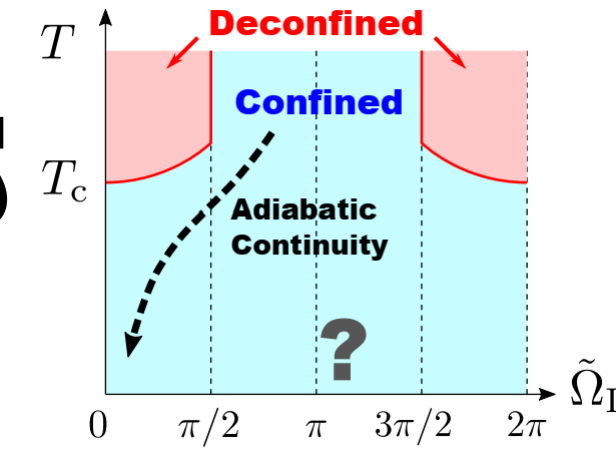
- Periodicity

$$\hat{\rho}(\tilde{\Omega}_I + 2\pi, \theta_q) = \hat{\rho}(\tilde{\Omega}_I, \theta_q + \pi),$$

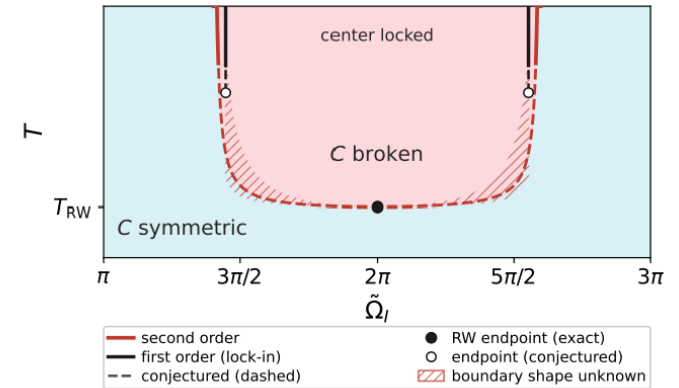
- Or

$$\hat{\rho}(2\pi, 0) = \hat{\rho}(0, \pi),$$

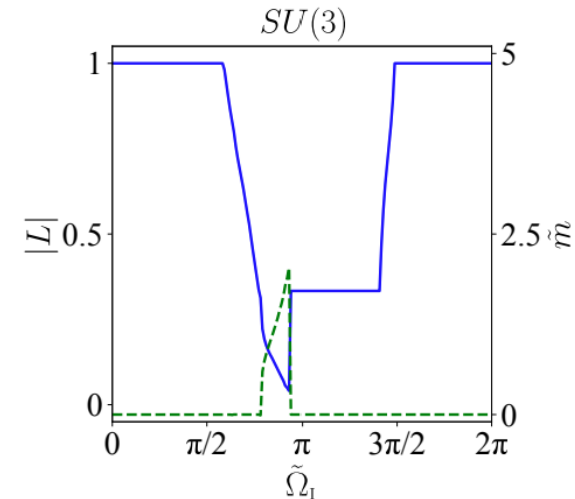
- For SU(3) with imaginary chemical potential $\theta_q = \pi$ is a Roberge-Weiss point
- Thus, $\tilde{\Omega}_I = 2\pi$ is exactly mapped to an RW point.



Shi Chen, Kenji Fukushima, Yusuke Shimada,
Phys.Rev.Lett. 129 (2022) 24, 242002



Rin Takada, 2608.25849



Shi Chen, Kenji Fukushima, Yusuke Shimada,
Phys.Lett.B 859 (2024) 139107

CHROMOMAGNETIC CONDENSATE

- Another well known gluon condensate is chromomagnetic condensate

G.K. Savvidy, Phys. Lett. B, 71:133, 1977

- Although the constant chromomagnetic configuration is not stable

N.K Nielsen and P. Olesen., Nucl. Phys. B, 144(2-3):376–396, 1978.

- Asymptotic freedom can be understood in term of vacuum polarizability

$$E_{\text{vac,QCD}} = -\frac{1}{2} V H^2 \frac{(33 - 2N_F) q^2}{48\pi^2} \log \frac{\Lambda^2}{|gH|}$$



QCD Beta function

$$\beta(g) = -\frac{(33 - 2N_F)g^3}{48\pi^2}$$

Asymptotic freedom as a spin effect

N. K. Nielsen^{a)}

Fysisk Institut, Odense University, Odense, Denmark

(Received 25 August 1980; accepted 26 November 1980)

It is shown how both the qualitative and the quantitative features of the asymptotic freedom of quantum chromodynamics can be understood in a rather intuitive way. The starting point is the spin of the gluon, which because of the gluon self-coupling makes the vacuum behave like a paramagnetic substance. Combining this result with Lorentz invariance, we conclude that the vacuum exhibits dielectric antiscreening and hence asymptotic freedom. The calculational techniques are with some minor modifications those of the Landau theory on the diamagnetic properties of a free-electron gas.

- It is interesting to see how rotation affects polarized gluon field(chromomagnetic condensate)

SU(2) YANG-MILLS THEORY UNDER ROTATION

- We include both Polyakov loop and chromomagnetic field by background field

$$\bar{A}_\mu^3 = (\phi, \frac{1}{2}Hy, -\frac{1}{2}Hx, 0) \quad \text{H along the rotation axis}$$

- As an extension of Kenji's work, we consider imaginary rotation and $r=0$

$$V(r=0) = \frac{1}{2}H^2 + \frac{gH}{2\pi\beta} \sum_{n=-\infty}^{\infty} \sum_{\lambda=0}^{\infty} \sum_{s=\pm 1} \int \frac{dk_z}{2\pi} \ln[(\omega_n + s\Omega_I + g\phi)^2 + gH(2\lambda + 1 + 2s) + k_z^2]$$

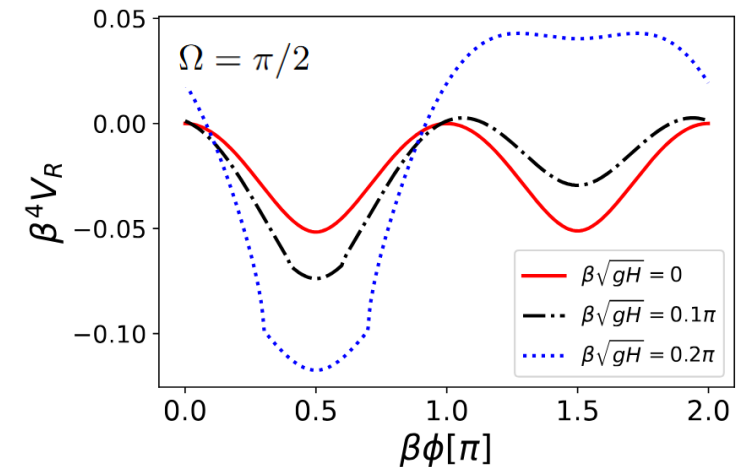
Tachyonic mode
when $\lambda = 0, \quad s = -1$

- The effective potential

$$V_R = \frac{11g^2H^2}{48\pi^2} \ln\left(\frac{gH}{\mu_0^2}\right) - \frac{(gH)^{\frac{3}{2}}}{\pi^2\beta} \sum_{n=1}^{\infty} \frac{1}{n} [K_1(n\beta\sqrt{gH}) - \frac{\pi}{2}Y_1(n\beta\sqrt{gH})] \cos n(\tilde{\phi} - \tilde{\Omega}_I)$$

$$- 2\frac{(gH)^{\frac{3}{2}}}{\pi^2\beta} \sum_{n=1}^{\infty} \sum_{\lambda=0}^{\infty} \frac{1}{n} \sqrt{2\lambda + 3} K_1(n\beta\sqrt{gH(2\lambda + 3)}) \cos n\tilde{\phi} \cos n\tilde{\Omega}_I$$

$$V_I = -\frac{(gH)^2}{8\pi} - \frac{(gH)^{\frac{3}{2}}}{2\pi^2\beta} \sum_{n=1}^{\infty} \frac{1}{n} J_1(n\beta\sqrt{gH}) \cos n(\tilde{\phi} - \tilde{\Omega}_I)$$



GLUON CONDENSATE

- Minimize the real part of effective potential at high temperature

$$\frac{\partial V_R}{\partial(gH)} = \frac{\partial V_R}{\partial(g\phi)} = 0$$

- We focus on real part
- For small imaginary rotation, chromomagnetic condensate increases

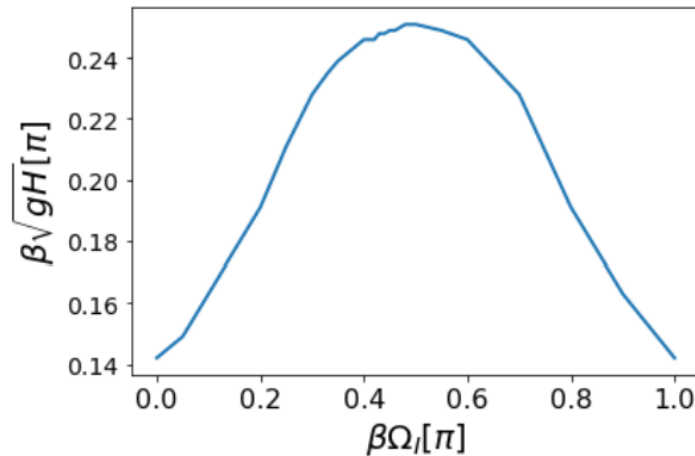


FIG. 3. Chromomagnetic condensate as a function of imaginary angular velocity at $T = 10\mu_0$.

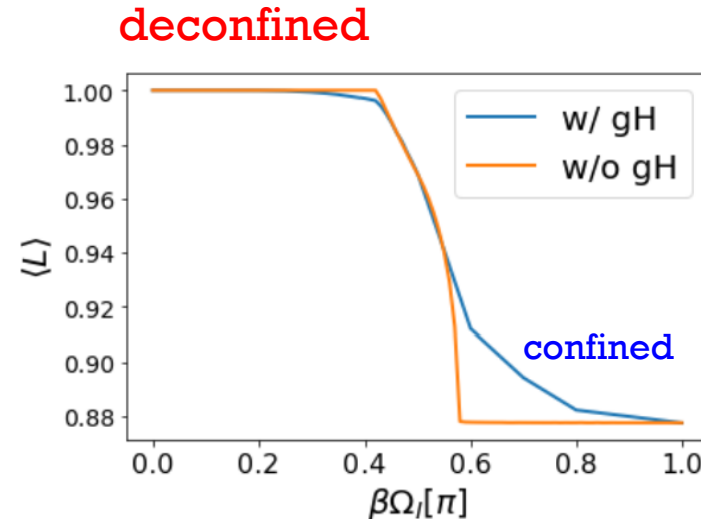


FIG. 6. Ployakov loop at $T = 10\mu_0$.

EFFECTIVE COUPLING CONSTANT & MOMENT OF INERTIA

HLC, Xu-Guang Huang, Phys.Rev.D 114 (2026) 1, 014005

- We can extract effective coupling constant

$$g_{\text{eff}}^{-2}(T, \Omega_I) = g(\mu)^{-2} + \left[-\frac{11}{12\pi^2} \left(\ln \frac{\beta\mu}{4\pi} - \gamma \right) + \frac{7+4C_1}{16\pi^2} - \frac{11}{48\pi^4} \zeta(3) \beta^2 \Omega_I^2 \right].$$

- The coupling increase with imaginary rotation: Tc increases with imaginary rotation
- Analytic continuation: Tc decreases with real rotation

In consistent with previous model studies

- The chromomagnetic field contribute negatively to moment of inertia

$$I = -\frac{\partial^2 V_R}{\partial (i\Omega_I)^2} = \frac{1}{\beta^2} \left[1 - \frac{11-12C_3}{12\pi} \beta^2 gH - \frac{11}{48\pi^4} \zeta(3) (\beta^2 gH)^2 \right].$$

negative Barnett effect found in lattice simulation

Braguta, et.al (2024)

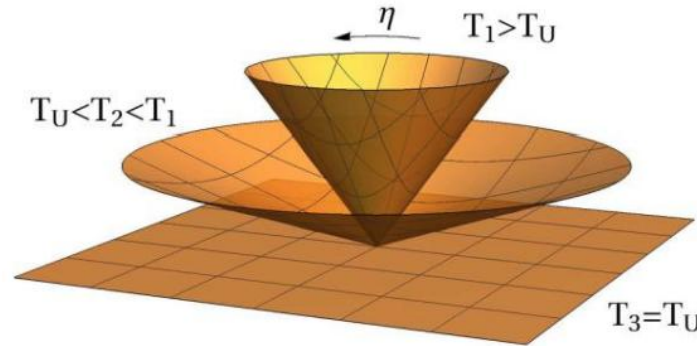
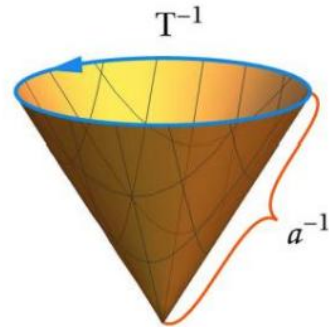
ACCELERATION

RINDLER COORDINATE

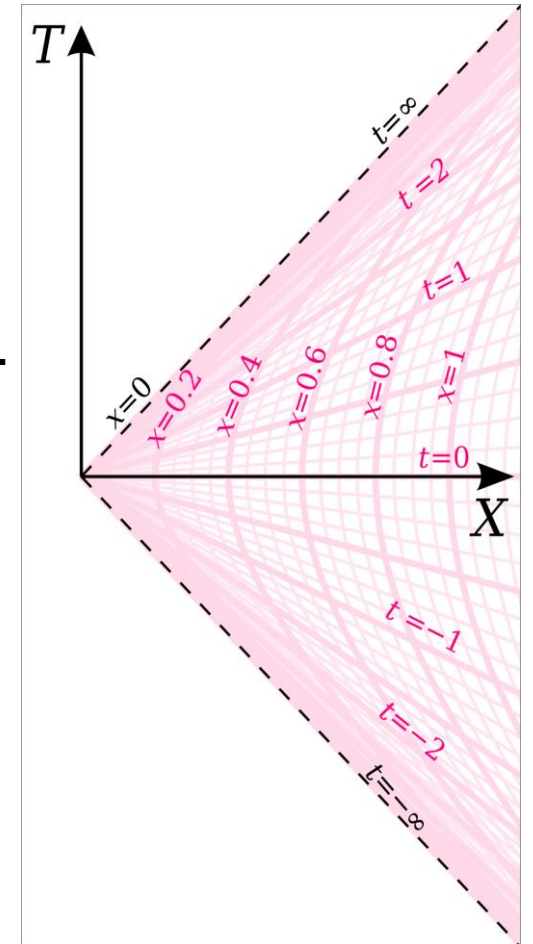
$$ds^2 = -(\alpha x)^2 dt^2 + dx^2 + dy^2 + dz^2$$

- Euclidean Rindler $dt \rightarrow -id\tau$, $\tau \in [0, T_0^{-1})$

T_0 : global temperature;
 $T_{loc} = T_0/az$:
 local Tolman temperature.



- Unruh temperature $T_U = \frac{\hbar a}{2\pi c k_B} \approx 4.06 \times 10^{-21} \text{ K} \cdot \text{s}^2 \cdot \text{m}^{-1} \times a$, $\longleftrightarrow$ Hawking temperature $T_H = \frac{\hbar g}{2\pi c k_B}$



- Euclidean Rindler at Unruh temperature is equivalent to Minkowskian vacuum!

UNRUH EFFECT

- Two vacuum

Minkowski vacuum

$$a_k^M |0_M\rangle = 0$$

Rindler vacuum

$$a_k^R |0_R\rangle = 0$$

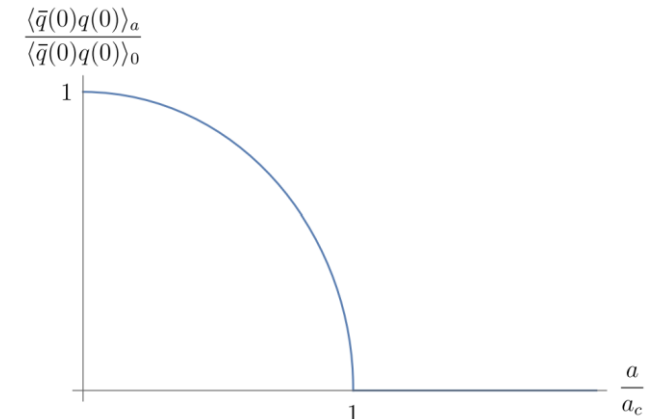
- Bogoliubov transformation

$$a_k^R = \left[2 \sinh \left(\frac{\pi \omega}{a} \right) \right]^{-1/2} \left(e^{\pi \omega / 2a} a_k^{(1)M} + e^{-\pi \omega / 2a} a_{-k}^{(2)M\dagger} \right)$$

- an observer who is uniformly accelerating through empty space (Minkowski vacuum) will perceive a thermal bath

$$\langle 0_M | a_k^{R\dagger} a_k^R | 0_M \rangle = \frac{1}{e^{\omega/T_U} - 1} \quad T_U = \frac{a}{2\pi}$$

We then expect,
for $T = a/2\pi$



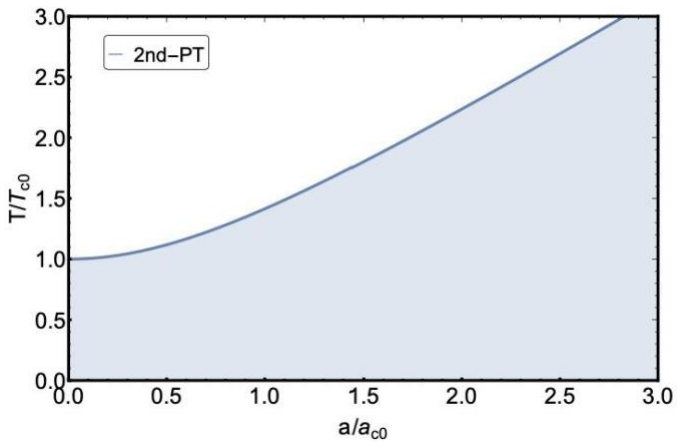
PHYSICAL REVIEW D 99, 125018 (2019)

QCD PHASE DIAGRAM UNDER ACCELERATION

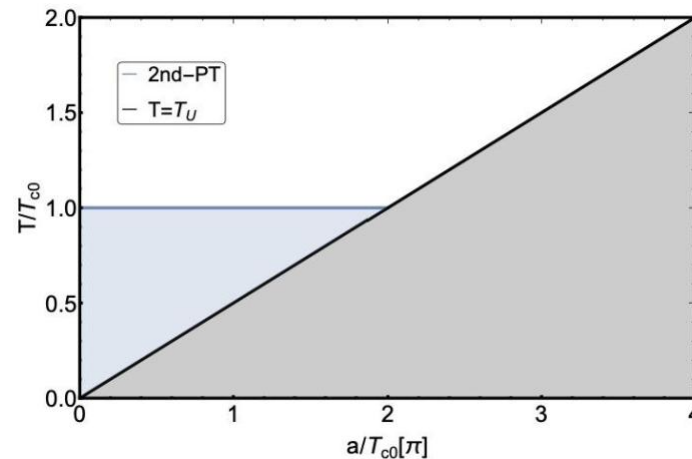
Zhi-Bin Zhu, HLC, Xu-Guang Huang, *Phys.Rev.D* 113 (2026) 3, 034005

- NJL model & Non-linear sigma model
- Dependence on regularization

Subtraction with respect to
the Minkowski vacuum

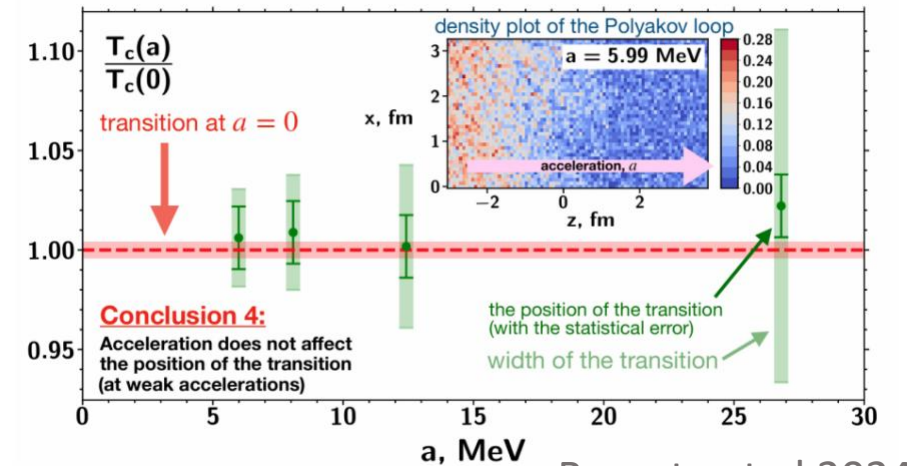


Subtraction with respect to
the Rindler vacuum



Lattice result

Phase diagram of gluon plasma under acceleration



Braguta et.al 2024

THERMODYNAMIC SUBTLETIES IN RINDLER SPACETIME

- To obtain the thermodynamic potential
- Usually, we compute the Euclidean partition function
- Its thermodynamic interpretation is straightforward in ultrastatic spacetimes
- Rindler spacetime is static, but not ultrastatic

Ultrastatic: $ds_E^2 = d\tau^2 + h_{ij}dx^i dx^j,$

Rindler: $ds_E^2 = (az)^2 d\tau^2 + dz^2 + d\mathbf{x}_\perp^2.$

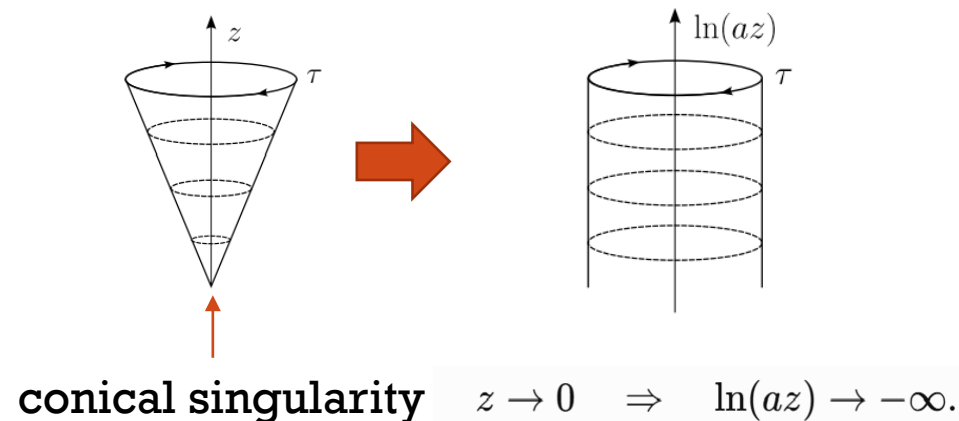
- Energy momentum tensor in Rindler spacetime $T_\mu^\nu = \text{diag}(\varepsilon, -p, -p, -p)$
- We have $\varepsilon - 3p = 0$. However, $\varepsilon \neq -\partial_{\beta_0}(\beta_0 p)$
- The local effective potential V_{ER} obtained directly from the Rindler Euclidean path integral is simply $-p$

OPTICAL METRIC

- Another useful method for computing one-loop quantities is the optical-metric approach.

$$ds^2 = g_{00}(x) d\tau^2 + g_{ij}(x) dx^i dx^j \quad \xrightarrow{\text{Conformal transformation}} \quad d\tilde{s}^2 = d\tau^2 + \frac{g_{ij}(x)}{g_{00}(x)} dx^i dx^j$$

- For Rindler spacetime, the conformal transformation leads to $S^1 \times H^3$
- Light rays follow geodesics of the optical spatial geometry
- The optical geometry is ultrastatic, with a uniform temperature T_0



EXAMPLE : MASSLESS FERMION

- In Euclidean Rindler:

$$V_{f,T}^{\text{ER}} = \frac{7\pi^2}{180 z^4 (\beta_0 a)^4} + \frac{1}{72 z^4 (\beta_0 a)^2}$$

energy density

$$\epsilon = \langle T^t_t \rangle = 3V_{f,T}^{\text{ER}}$$

- We find thermodynamic relation

$$\epsilon = \frac{\partial}{\partial \beta_0} (\beta_0 f_{\text{loc}})$$

- So we identify V_{opt} with the physical thermodynamic potential after the Weyl transformation.

In optical metric

$$V_{f,T}^{\text{opt}} = \frac{7\pi^2}{180 \beta_0^4} + \frac{a^2}{24 \beta_0^2}$$

If we define free energy as

$$f_{\text{loc}} = - e^{4\sigma} V_{f,T}^{\text{opt}}$$

GENERAL RELATION

- We prove that this relation also holds for scalar fields, massive spinors, and gauge fields.
- In even dimension D

$$\epsilon = \frac{\partial}{\partial \beta} (\beta f_{\text{loc}}) = -e^{D\sigma} \frac{\partial}{\partial \beta} (\beta V_T^{\text{opt}})$$

- We should minimize V_{opt} when we consider Polyakov potential

$$\begin{aligned} V_{\text{opt}} &= V_{\text{gluon}} + V_{\text{ghost}} \\ &= T_0^4 \sum_{\alpha} \left[\frac{2\pi^2}{3} B_4 \left(\frac{\phi \cdot \alpha}{2\pi} \right) - \tilde{a}^2 B_2 \left(\frac{\phi \cdot \alpha}{2\pi} \right) \right] \end{aligned}$$

- No periodicity as rotation case

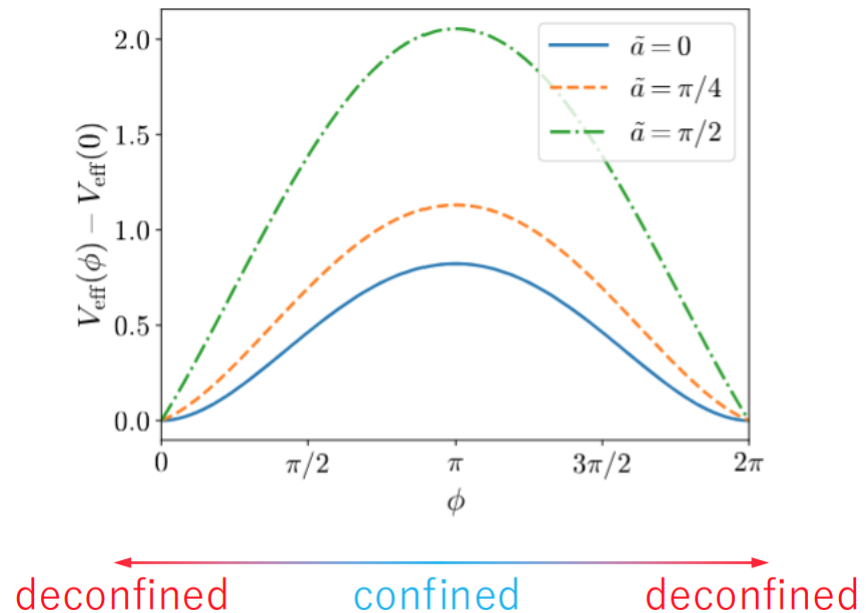
For comparison, in the rotating case,

$$V_{\Omega}|_{\tilde{r}=0} = \frac{2\pi^2 T_0^4}{3} \sum_{s=\pm 1} B_4 \left[\left(\frac{\phi + s\tilde{\Omega}_I}{2\pi} \right)_{\text{mod } 1} \right]$$

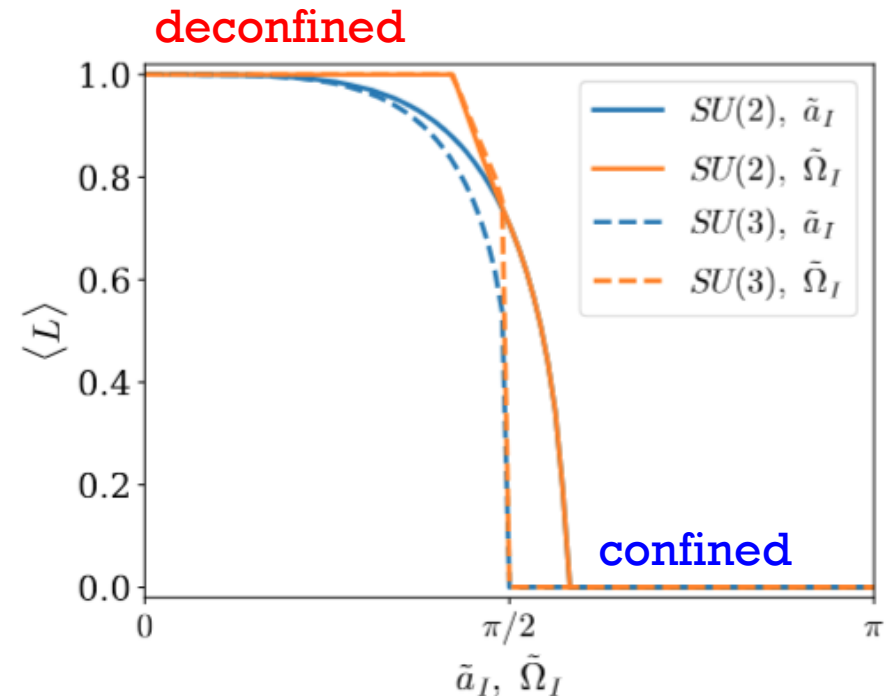
GLUON UNDER ACCELERATION

Phys.Rev.D 114 (2026) 5, 054012

- Acceleration prefers confinement



- Analogy between imaginary acceleration & imaginary rotation



HOW ABOUT ROTATION?

- Rotating metric is stationary

$$ds^2 = -(1 - \Omega^2 r^2)dt^2 + 2\Omega r^2 dt d\phi + dr^2 + r^2 d\phi^2 + dz^2.$$

- We can write

$$ds^2 = -N^2(dt + \underbrace{A_i dx^i}_{\text{red circle}})^2 + h_{ij}dx^i dx^j.$$

- A_i should be treated like gauge field Dmitri V. Fursaev, Class.Quant.Grav. 19 (2002) 3635-3652
- Works well for S^3 , but does not yield expected result for rotating metric
- The radial coordinate r seems crucial
- For massless fermion

$$T \sum \ln [1 + e^{-\beta(E - \Omega j)}] \rightarrow P_{\text{eff}} = \frac{7\pi^2}{180} T^4 + \frac{3\omega^2 + a^2}{72} T^2$$

Expected to be our V_{ER}
which satisfies $e = 3p$



$$P = \frac{7\pi^2}{180} T^4 + \frac{\omega^2 + a^2}{24} T^2$$

Expected to be our V_{opt}
which satisfies thermodynamic relation

Here a comes from rotation (centrifugal)
F. Becattini and R. Singh, Eur. Phys. J. C 85, 1338(2025)
V.E. Ambrus and A. Geci'c, 2509.17640

OUTLOOK

- What can we learn about rotation from acceleration?
- Include fermionic effects in accelerated systems
- Higher-loop corrections
- Clarify the relations among $V_{ER}, V_{opt}, T_0^0, P \dots$

THANKS